WEIGHT AND WEIGHTLESSNESS

Weight is simply the force of gravity (do you remember the formula $F_g = mg$?).

We are not normally aware of this force – what we are actually aware of is the normal reaction force that acts on us when we are in contact with, say, the floor or the seat on which we sit. Usually, this normal reaction force is equal to our weight (giving a net force of zero, which is why you do not accelerate while sitting on a chair). There are situations, however, when the normal reaction force is not equal to the weight. Such as?

EXAMPLE 11

A physicist of mass 60 kg is patiently standing in an elevator. Determine the magnitude of the normal reaction force that acts on the physicist when the elevator is:

(a) Stationary.

(b) Moving upwards with a constant speed of 2 m/s.

(c) Accelerating upwards at 2 m/s^2.

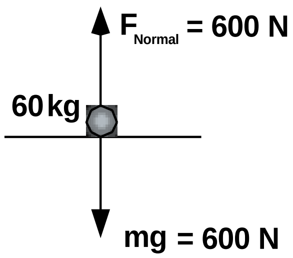
(d) Accelerating downwards at 2 m/s^2.

Remember that, in each case, the reaction force on the violinist is her *apparent* weight.

Solution

(a) The key fact here is not that the velocity is zero, but rather that the acceleration is zero presumably, the elevator is remaining stationary). This implies that $F_{net} = 0$, which in turn implies that the magnitude of the normal reaction force must be equal to the weight.

Now, the weight, mg, is 600 N and therefore the normal reaction force is also 600 N. Since force is a vector, we must include direction in our answer. So, normal reaction force = 600 N up.



(b) As in part (a), the velocity is constant, so a = 0. Thus, again, the normal reaction force is 600 N up.

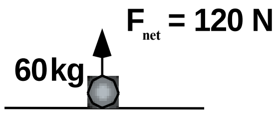
(c) Since there is now an acceleration (specifically, a = 2 m/s^2 up), we know that a net force must be acting on the violinist. Finding this is easy:

$$F_{net} = ma$$

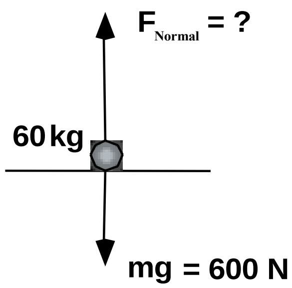
$$F_{net} = 60 \times 2$$

$$F_{net} = 120\ N\ up$$

(Notice that the net force is up, for the formula $F_{net} = ma$ tells us that F_{net} is always in the same direction as the acceleration.)



The net force is responsible for the acceleration. The net force is the sum of two forces: The weight and the normal reaction force. These forces are shown in the next diagram. Notice that the vector representing the normal reaction force is drawn longer than the vector representing the weight force. The normal reaction force must be greater than the weight in order to give an upward net force.



In fact, the normal reaction force must be 120 N greater than the weight force. Since the weight is 600 N, the normal reaction force must be 720 N upwards.

Alternative Solution

Some of you will not particularly like the above method. For those stricken with a love for algebra, try this:

First, just as we did above, use $F_{net} = ma$ to find the net force. Then note that the net force is the sum of all forces (two forces in this case) that act on the object.

$$F_{net} = mg + F_{Normal}$$

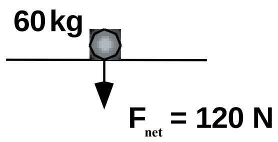
$$+120 = (60)(-10) + F_{Normal}$$ (Note our sign convention: up = +, down = -)

$$+120 = -600 + F_{Normal}$$

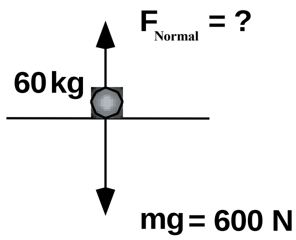
$$F_{Normal} = +720$$

Since + = up, our answer is 720 N up.

(d) Now a = 2 m/s^2 down. We now find that $F_{net} = 120\, N\ down$.



In order to produce a net force that is directed downwards, the normal reaction force must be less than the weight.



Specifically, the normal reaction force must be 120 N less than the weight, giving us an answer of 480 N up.

Alternative Solution

$$F_{net} = mg + F_{Normal}$$

$$-120 = (60)(-10) + F_{Normal}$$

$$-120 = -600 + F_{Normal}$$

$$F_{Normal} = +480$$

$$F_{Normal} = 480\, N\ up$$

MOMENTUM AND IMPULSE

Momentum, $p = mv$, is a *vector quantity* (because it is the product of a vector, v, and a scalar, m). The direction of the momentum vector is the same as the direction of the velocity vector.

Momentum is terribly interesting because it is conserved – see below.

Consider the equation, $F_{net} = ma$. Since, $a = \frac{\Delta v}{\Delta t}$, we can rewrite $F_{net} = ma$ as:

$$F_{net} = m\frac{\Delta v}{\Delta t}$$

We can now transpose the equation thus: $F_{net}\Delta t = m\Delta v$.

The quantity $F_{net}\Delta t$ is known as the *impulse* (unit: Newton second, N s). Impulse is simply force multiplied by the time over which the force acts. Impulse is a vector quantity – it has the same direction as the force.

The quantity $m\Delta v$ is the *change in momentum*, i.e. $\Delta p = m\Delta v$.

Δp (the change in momentum) has the same direction as Δv (the change in velocity).

The equation $F_{net}\Delta t = m\Delta v$ tells us that **impulse = change in momentum**.

It should be clear that an impulse will indeed cause a change in momentum: If a force acts over any period of time, it will cause an acceleration, i.e. a change in velocity. If an object's velocity changes, then it follows that its momentum also changes.

Note also that if we write the equation $F_{net} = m\frac{\Delta v}{\Delta t}$ thus: $F_{net} = \frac{m\Delta v}{\Delta t}$.

It follows that: $F_{net} = \frac{\Delta p}{\Delta t}$

Net force = Rate of change of momentum

This is simply Newton's 2nd Law.

As one would expect, Newton's 2nd Law implies that large forces will cause a rapid change in momentum.

EXAMPLE 12
A billiard ball of mass 0.10 kg approaches the cushion at 6.0 m/s and bounces off at 4.0 m/s in the opposite direction. The ball is in contact with the cushion for 0.05 s.

Find:

(a) The force applied by the ball on the cushion.

(b) The impulse given to the ball by the cushion.

Solution

We begin by choosing a suitable definition of directions, say:

Towards cushion = positive Away from cushion = negative

Thus, for the ball, u = +6.0 m/s and v = -4.0 m/s

(a) By Newton's 3rd Law,

(Force applied by ball on cushion) = - (Force applied by cushion on ball)

i.e. $F_c = -F_b$

Calculating the force on the ball is easy:

$$F_b = \frac{\Delta p}{t} = \frac{m\Delta v}{t} = \frac{m(v-u)}{t}$$

$$F_b = \frac{0.10(-4.0 - +6.0)}{0.05}$$

$$F_b = -20N$$

But $F_c = -F_b = -(-20)$

So $F_c = +20N$

$F_c = 20N$ towards the cushion

(b) Impulse $= \Delta p$
$= m\Delta v$
$= m(v-u)$
$= 0.10(-4.0 - +6.0)$
$= 1.0Ns$

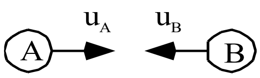
QUESTION 18
Explain how a soft playground surface can minimise the extent of injuries experienced by a child that falls over.

Solution

CONSERVATION OF MOMENTUM

Momentum is conserved. This means that in any *closed* system, the *total* momentum always remains *constant*. (By a closed, or isolated, system we mean one that is not subjected to any external forces.) Momentum can be transferred from object to object (meaning that individual objects within the closed system may gain or lose momentum), but the sum of the momenta of all the objects never changes.

Consider two objects, A and B, moving towards each other:



When these objects collide, Newton's 3rd law tells us that the force that A exerts on B will be equal in magnitude but opposite in direction to the force that B exerts on A, i.e.

$F_A = -F_B$

But, Newton's 2nd Law states that, $F = \frac{\Delta p}{\Delta t}$, so we could write:

$$\frac{\Delta p_A}{\Delta t} = -\frac{\Delta p_B}{\Delta t}$$

$$\Delta p_A = -\Delta p_B$$

$$m_A \Delta v_A = -m_B \Delta v_B$$

$$m_A (v_A - u_A) = -m_B (v_B - u_B)$$

$$m_A v_A - m_A u_A = -m_B v_B + m_B u_B$$

$$m_A v_A + m_B v_B = m_A u_A + m_B u_B$$

This is simply stating that (final momentum of A + the final momentum of B) = (initial momentum of A + initial momentum of B), i.e. total momentum before collision = total momentum after collision.

In collisions, individual objects can gain or lose momentum. But the only way that the momentum of an object can change is if momentum is transferred from one object to another.

Remember: **Total** momentum in the system before the collision =
Total momentum in the system after the collision.

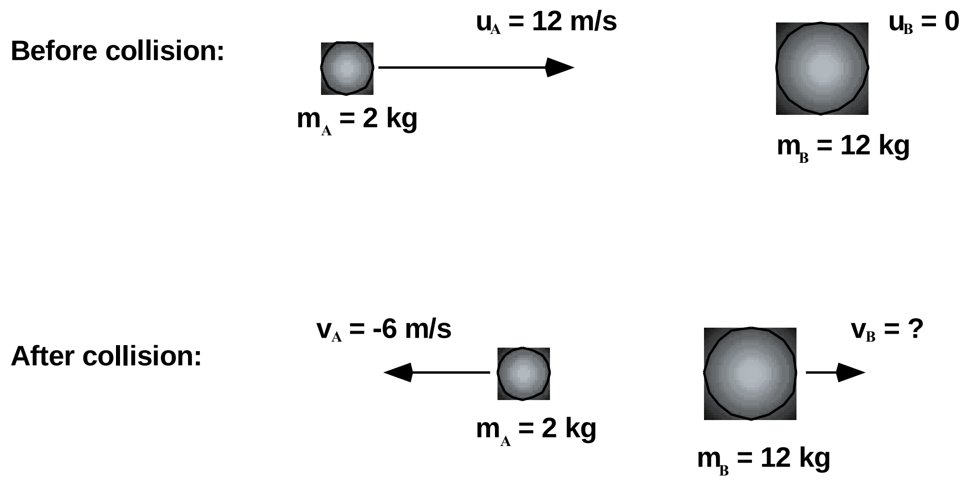
So, in a system with only two objects:

Momentum lost by one object = Momentum gained by the other object.

EXAMPLE 13
Object A, which has a mass of 2 kg and is moving with a speed of 12 m/s, collides with a Object B, which is stationary and has a mass of 12 kg. As a result of the collision, Object A rebounds with a speed of 6 m/s. With what speed does Object B move off?

Solution

Yes, we begin with suitable diagrams.



We need to adopt a suitable sign convention, so we shall take the original direction of motion of Object A to be the positive direction.

Total momentum before collision = Total momentum after collision

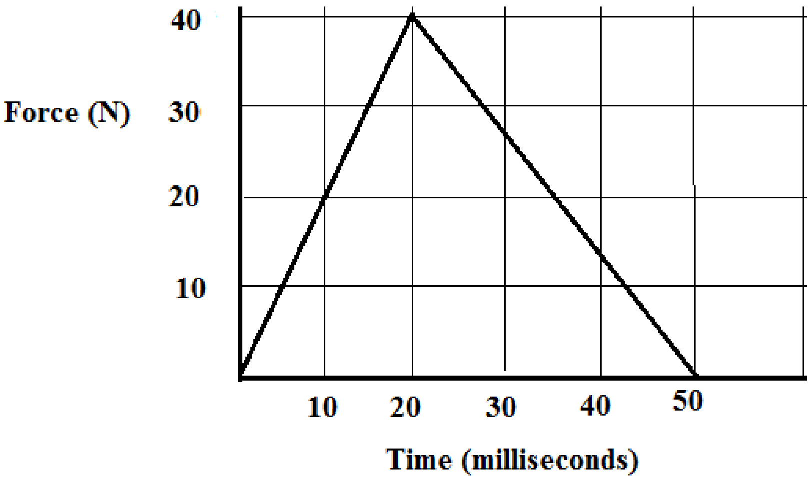
$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$
$$(2)(+12) + 0 = (2)(-6) + 12v_B$$
$$24 = -12 + 12v_B$$
$$36 = 12v_B$$
$$v_B = +3m/s$$

That's all there is to it. But let us do a simple check:

Object A had +24 kg m/s of momentum before the collision. It had -12 kg m/s of momentum after the collision. Therefore Δp for Object A is -36 kg m/s (i.e. final momentum - initial momentum). The Law of Conservation of Momentum tells us then that Δp for Object B must be +36 kg m/s. Since Object B initially had zero momentum, it must end up travelling at +3 m/s in order to have a final momentum of +36 kg m/s.

EXAMPLE 14

During a squash game, the ball of mass 40 g is served. Assume that the initial speed of the ball is zero. The racquet exerts a force on the ball as shown in the diagram below:



(a) What is the maximum force that the racquet exerts on the ball?

(b) What is the maximum force that the ball exerts on the racquet?

(c) Calculate the net impulse delivered to the ball.

(d) What is the change in momentum of the ball?

(e) Calculate the speed with which this ball was served.

Solution

(a) 40 N (read directly from the graph)

(b) 40 N (The maximum force exerted by the racquet on the ball equals the maximum force exerted by the ball on the racquet.)

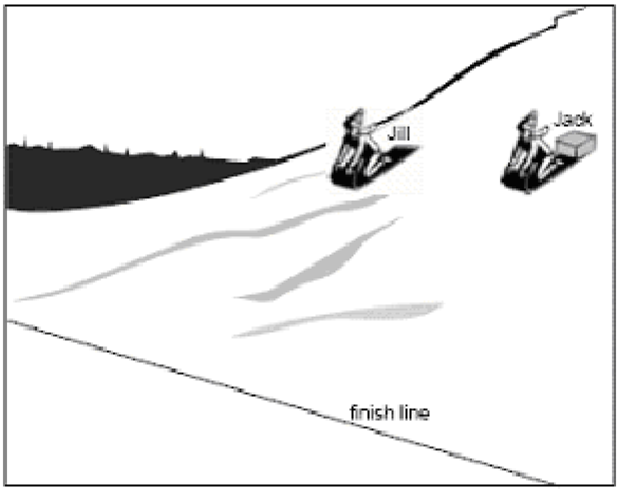
(c) Impulse can be determined from the area under the graph. Note the time units are milliseconds which must be converted to seconds. Area = 1.0 Ns

(d) The change in momentum = impulse, i.e. 1.0 Ns

(e) $\Delta P = m\Delta v$

Hence $\Delta v = \frac{\Delta P}{m} = \frac{1.0}{0.04} = 25ms^{-1}$

QUESTION 19
Jack and Jill are racing their toboggans down an icy hill. Jack and Jill are of similar mass and are using the same type of toboggan. When Jack is a certain distance from the end of the race they are travelling with the same velocity. Jack is behind Jill and decides that if he is going to win the race he must lighten his toboggan, so he pushes a box containing their ice-skating gear off the side of his toboggan.



Explain giving reasons, whether this will be a successful way for Jack to catch up to Jill and help him win the race.

Solution

ELASTIC AND INELASTIC COLLISIONS

In any closed system, the total momentum and the total energy remain constant. There are some important differences, however, between momentum and energy. One difference is that momentum is a vector while energy is not. Another difference is that energy, unlike momentum, comes in a variety of types. So while it is true that the total energy in a closed system never changes, there is no reason why the type of energy cannot change. This means that, say, kinetic energy need not be conserved. Indeed, in collisions, kinetic energy is rarely conserved – it is readily converted to other types of energy.

We define two types of collisions:

- Elastic collisions, in which kinetic energy is conserved.
- Inelastic collisions, in which kinetic energy is not conserved.

Note that while kinetic energy need not be conserved in certain interactions, two fundamental principles always apply:

- Law of Conservation of Momentum.
- Law of Conservation of Energy.

EXAMPLE 15
If Object A, of mass 2 kg and moving at 12 m/s which collides with Object B, which is stationary and has a mass of 12 kg. After the collision, Object B is moving at 3 m/s while Object A is moving in the opposite direction at 6 m/s. Was this an elastic collision?

Solution

To determine whether the collision was elastic or inelastic, we need to calculate the total kinetic energy of the objects before the collision and compare it to the final kinetic energies of the bodies.

Initial kinetic energy = $\frac{1}{2}m_A u_A^2$ = 144 J

Final kinetic energy = $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$

= 36 + 54

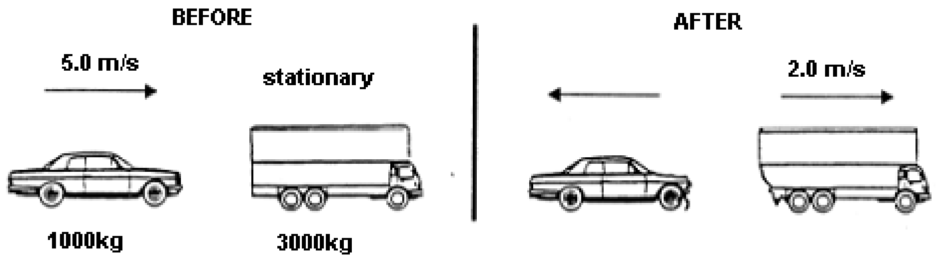
= 90 J

Since the final total kinetic energy of the system is less than the initial total kinetic energy the collision was inelastic.

A car of mass 1000 kg, travelling on a horizontal road with a speed of 5.0 ms^{-1}, runs into the rear of a stationary truck of mass 3000 kg. Immediately after the collision the truck moves forward with a speed of 2.0 ms^{-1} and the car rebounds in the opposite direction.

In modelling this collision you should assume that:

- There is no driving force from either engine during the collision.
- No braking takes place during the collision.
- The car and truck remain in a straight line.



QUESTION 20
What is the speed of the car immediately after the collision?

Solution

QUESTION 21
This was an inelastic collision. Explain the meaning of the word inelastic and show, using calculations, that this collision was inelastic.

Solution

QUESTION 22
Where has the 'lost kinetic energy' gone?

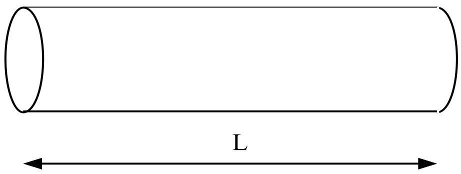
Solution

QUESTION 23
The car and truck were in contact for 20 ms (0.02 s). What was the magnitude of the average force on the car during the collision?

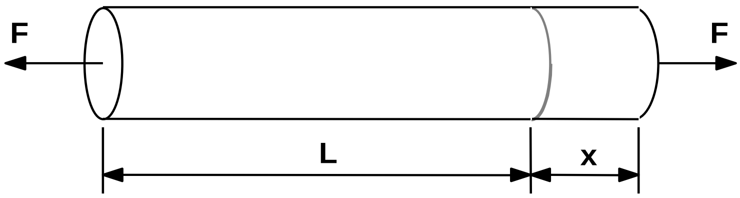
Solution

ELASTIC POTENTIAL ENERGY

All objects can be extended or compressed if a suitable force is applied. If the object is left alone, its length is called the natural length, denoted by L on the following diagram.



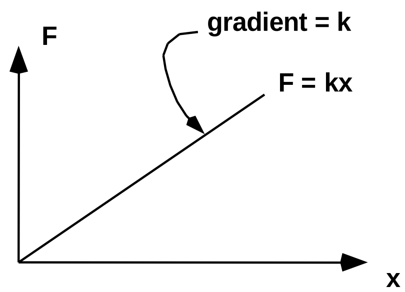
An applied force, however, causes the object to change its length by an amount x



The quantity x is called the extension (or compression, if a compressive force is applied).

HOOKE'S LAW

For many objects, the force required to extend the object is directly proportional to the required extension. A graph of F vs. x would thus be linear:

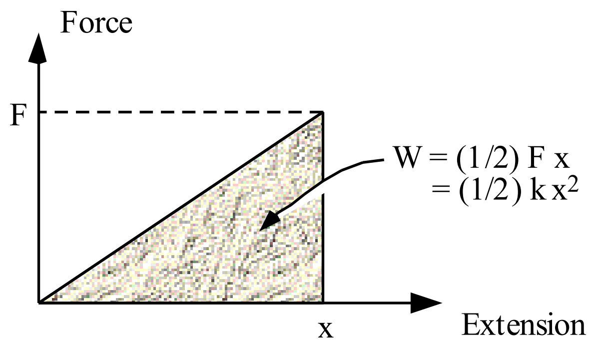


The equation of the line is $F = kx$, where k is the gradient. This equation is known as Hooke's Law. Be very careful: not all objects obey Hooke's Law.

The gradient, k, is known as the *force constant*. Notice that $k = \frac{F}{x}$.

This tells us that the units of k are N/m. If we know k, we know how easy or difficult it is to extend an object, i.e. we know how many newtons of force we need for each metre of extension.

In applying a force over a distance, one is doing work. Since the force is not constant, we can calculate the work done by using the equation $W = F.d$ only if we use the average force (and this is a perfectly fine method). In what amounts to the same thing, we could calculate the work done by finding the area under the graph.



Thus: $W = \frac{1}{2} F x$

But, since $F = kx$, we could substitute this into the above equation to obtain $W = \frac{1}{2} k x^2$.

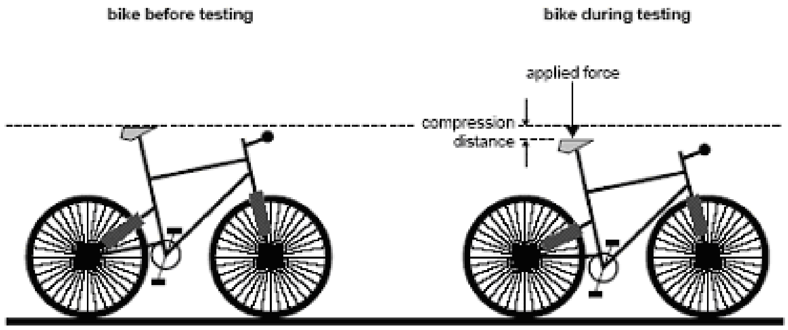
Of course, since energy is conserved, the work done in extending the object cannot just disappear – it is stored in the object as elastic potential energy U_e.

The above equations can therefore be written as:

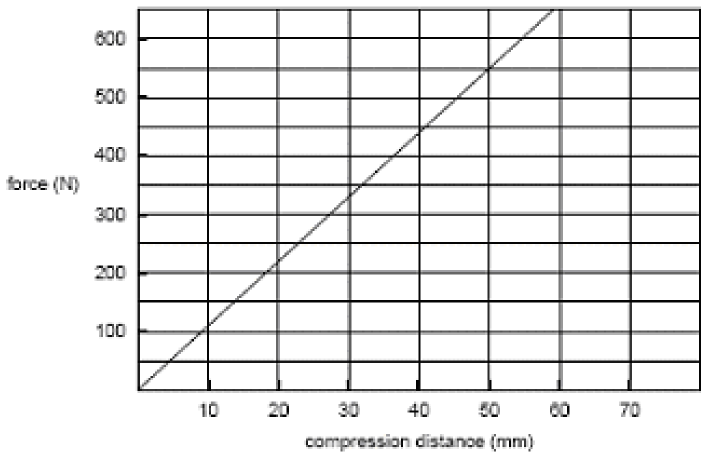
$$U_e = \frac{1}{2} F x \qquad \text{and} \qquad U_e = \frac{1}{2} k x^2$$

If an object does not obey Hooke's Law, these formulae cannot be used. Instead, elastic potential energy is calculated simply by finding the area under the graph.

Mountain bikes are often made with front and rear springs to provide a smoother ride.



A manufacturer investigates the compression of a bike by pushing down on the seat and measuring how far the seat moves down as a result. The result of these investigations is in the graph below showing the vertical force on the seat versus compression distance.



QUESTION 24

Calculate the mass of a person who produces a compression distance of 50 mm when seated on this mountain bike. Use $g = 9.8\ Nkg^{-1}$.

Solution

QUESTION 25

Calculate the total potential energy stored in the springs when the person is seated on the mountain bike.

Solution