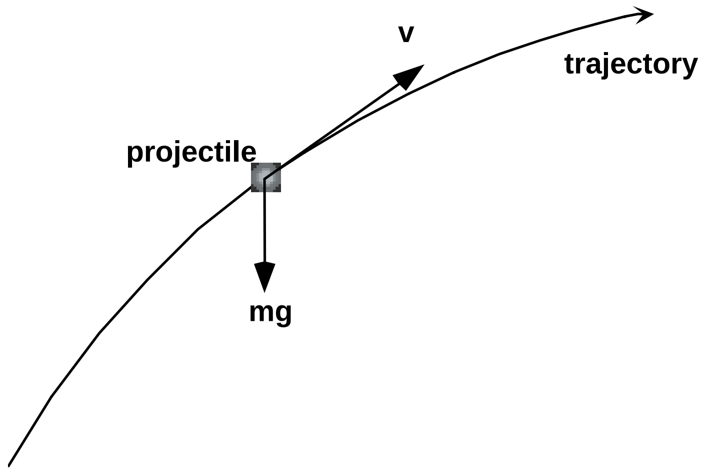
PROJECTILE MOTION

A projectile is any object that has been thrown through the air. A force must necessarily set the object in motion initially but, while it is moving through the air, no force other than gravity acts on it (we shall ignore air resistance for now).

Thus, a brick can be a projectile but a rocket or an airplane cannot.

The path, or trajectory, of the projectile is curved (it is, in fact, parabolic).

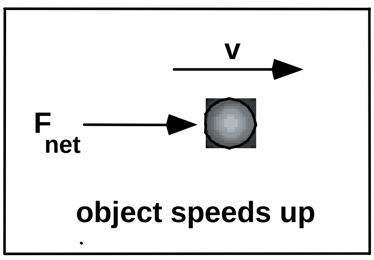


At any given point in the motion, the velocity vector is always a tangent to the path.

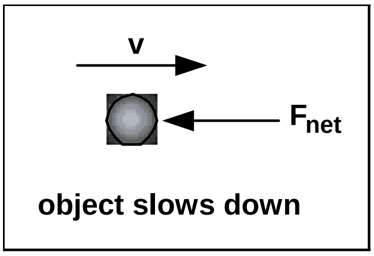
Note also that the vector mg = weight force
= the only force acting on the object
= net force

Here is something to remember about net forces:

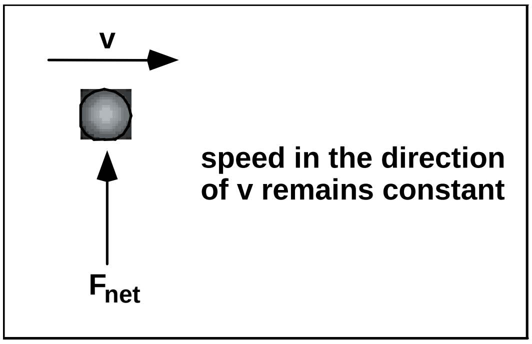
If the net force is in the same direction as the velocity of the object, the object speeds up.



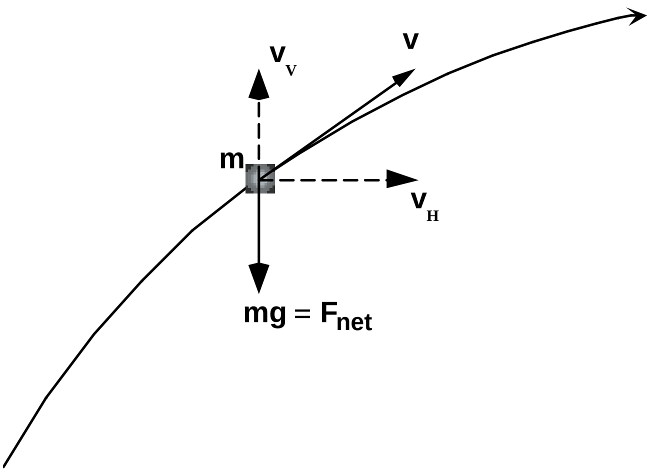
If the net force is in the opposite direction to the velocity of the object, the object slows down:



But, if the net force acts at right angles to the velocity vector, then the speed of the object in the direction of that vector does not change.



Now consider the following diagram:



Notice that the velocity vector, v, has two components:

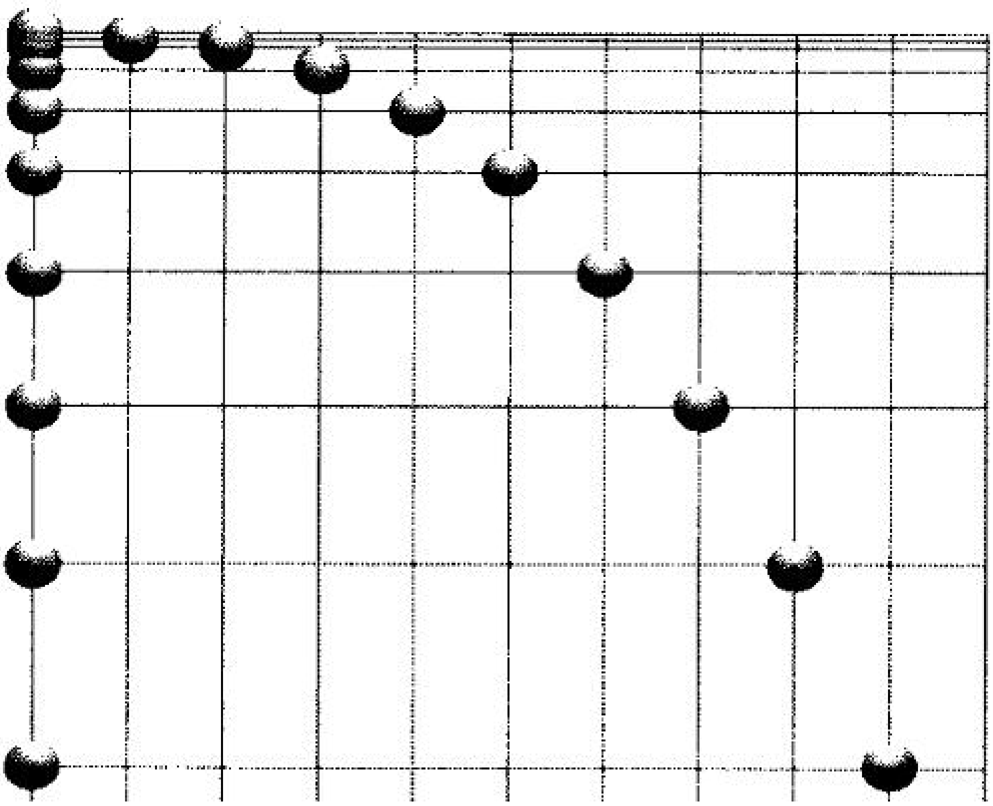
- A horizontal component, v_H.
- A vertical component v_V.

The net force, mg, has:

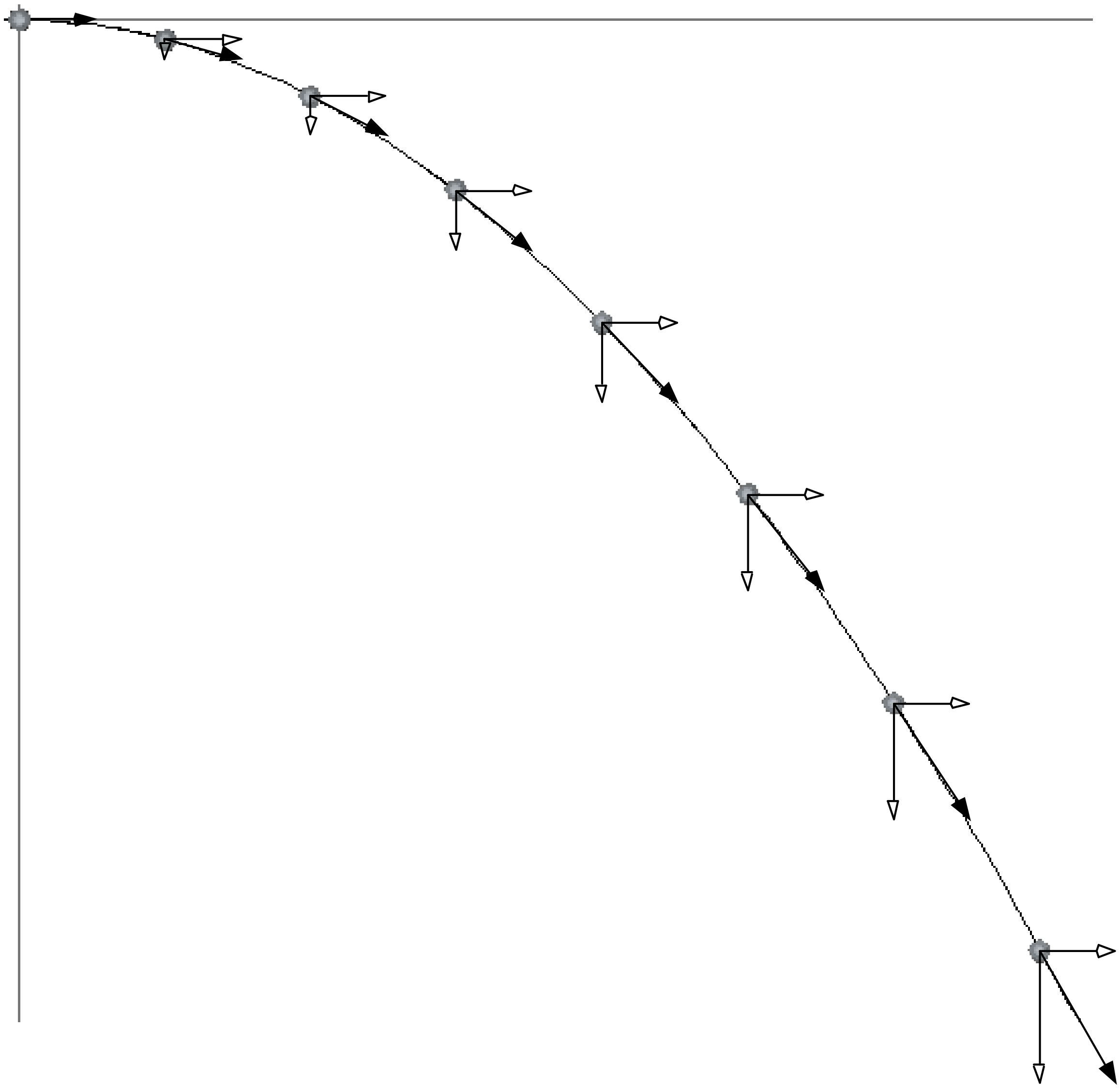
- An effect on v_V.
- No effect on v_H.

Thus, there is no acceleration horizontally (which is to say that v_H remains constant throughout the motion) but there is indeed a vertical acceleration.

The next diagram shows the path taken by a projectile that has been thrown horizontally. The position of the projectile is shown at equal time intervals. Notice that it travels at constant velocity horizontally (for it covers equal distances in equal time intervals) but it is accelerating vertically (it covers greater distances in successive equal time intervals). As one would expect, it is moving at constant speed horizontally, but it is speeding up vertically.



The diagram that follows shows how the velocity vector changes as the projectile moves along its trajectory. Also shown are the horizontal and vertical components of the velocity vector. Of course, the horizontal component stays constant but the vertical component changes.



We usually handle projectile motion problems by breaking up the motion into horizontal and vertical components.

For the horizontal component, we use:

$$v = \frac{d}{t}$$

For the vertical component, we use the constant acceleration formulae:

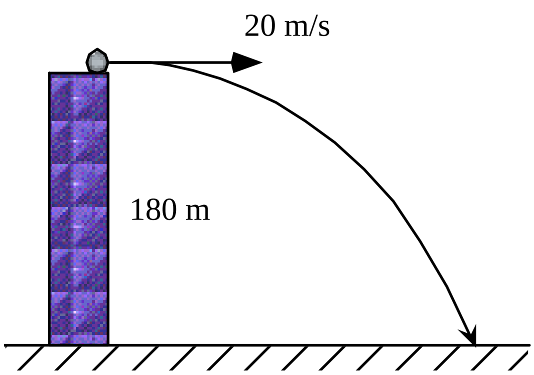
$$v = u + at$$

$$d = ut + \tfrac{1}{2}at^2$$

$$d = \left(\frac{u+v}{2}\right)t$$

$$v^2 = u^2 + 2ad$$

EXAMPLE 8
A dead water buffalo is thrown horizontally, with an initial speed of 20 ms^{-1} from the top of a 180 m high building.



(a) The time of flight.

(b) The horizontal distance from the base of the building to the point where the dead beast hits the ground.

(c) The velocity with which the carcass hits the ground.

Solution

(a) Here, we are only interested in the **vertical component**:

u = 0
a = -10 ms^{-2}
t = ?
d = -180 m

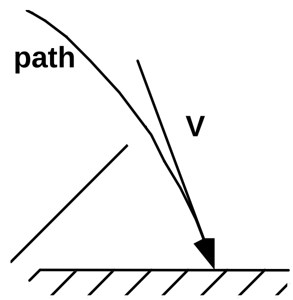
(i.e. if an object, initially at rest, accelerates at -10 ms^{-2}, at what time is its displacement -180 m?)

$$d = ut + \tfrac{1}{2}at^2$$
$$-180 = \tfrac{1}{2}(-10)t^2$$
$$t = 6\ s$$

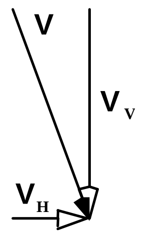
(b) Now we are only interested in the **horizontal component**:

$$v = \frac{d}{t}$$
$$20 = \frac{d}{6}$$
$$d = 120\ m$$

(c) At the point of impact, the critter is moving thus:



The vector, V, represents the final velocity. It is tangential to the path and has both horizontal and vertical components:



The horizontal component, V_H, is obviously 20 ms^{-1}. To find the vertical component, V_V, proceed thus:

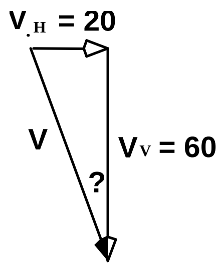
Vertical component:

u = 0
a = -10 m/s^2
v = ?
d = -180 m

$$v^2 = u^2 + 2ad$$
$$v^2 = 2(-10)(-180)$$
$$v = 60 \ m/s$$

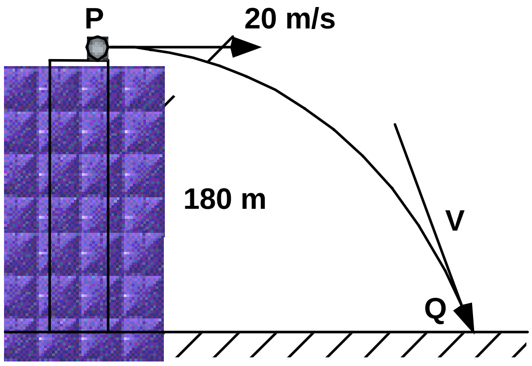
Or (this is better), knowing that t = 6 s, find v by using $v = u + at$.

We now know that V_V = 60 ms^{-1}. We can therefore draw the appropriate diagram, thus:



and use some simple trigonometry to find that V = 63 ms^{-1} at an angle of 18°26' to the vertical.

If asked to calculate the **speed** (rather than the velocity) with which the projectile strikes the ground, one could use the principal of conservation of energy, as follows:



Total energy at point Q = Total energy at point P.

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mgh$

$\frac{1}{2}v^2 = \frac{1}{2}u^2 + gh$

$\frac{1}{2}v^2 = \frac{1}{2}(20)^2 + (10)(180)$

$v = 63\ m/s$

No doubt you remembered much of this from Unit 2.

Kinetic energy: $E_k = \frac{1}{2}mv^2$

Gravitational potential energy: $U_g = mgh$

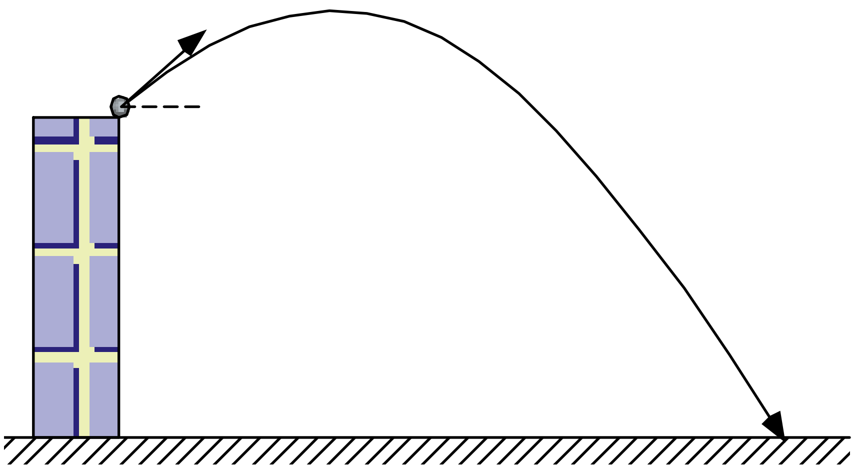
As well as the principle of energy being conserved – the total energy in a closed system always remains constant.

EXAMPLE 9
A dead flamingo is projected at an initial velocity of 100 ms^{-1} at an angle of elevation of 30° from the top of a 120 m high tower. Calculate:

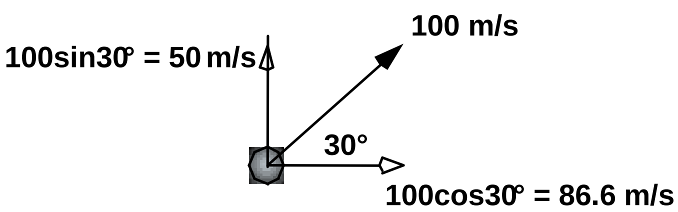
(a) The time taken to reach maximum height.
(b) The maximum height above ground reached.
(c) The time of flight.
(d) The horizontal distance from the base of the tower to the point where the feathered creature hits the ground.
(e) The velocity with which the critter hits the ground.

Solution

It would be rather nice to start with a diagram.



We will need to know the horizontal and vertical components of the initial velocity. Some simple trigonometry takes care of that:



(a) $u = +50\ m/s$

$a = -10\ m/s^2$

$t = ?$

$v = 0$ (Remember, at maximum height, the vertical component of the velocity is zero)

$v = u + at$

$0 = +50 + (-10)t$

$t = 5\ s$

(b) $u = +50\ m/s$

$a = -10\ m/s^2$

$d = ?$

$v = 0$

$v^2 = u^2 + 2ad$

$0 = 50^2 + 2(-10)d$

$d = 125\ m$

This is the height above the starting point. To find maximum height above the ground, add 120 m to obtain 245 m.

Alternative Solution

Total energy at maximum height = Total energy at starting point

$\frac{1}{2}mv^2 + mgh_{\max} = \frac{1}{2}mu^2 + mgh_{initial}$

$\frac{1}{2}v^2 + gh_{\max} = \frac{1}{2}u^2 + gh_{initial}$

$\frac{1}{2}(86.6)^2 + (10)h_{\max} = \frac{1}{2}(100)^2 + (10)(120)$ Note: v at max height = 86.6 m/s

$3750 + (10)h_{\max} = 5000 + 1200$

$h_{\max} = 245\ m$

(c) $u = +50\ m/s$

$a = -10\ m/s^2$

$t = ?$

$d = -120\ m$

$$-120 = 50t + \tfrac{1}{2}(-10)t^2$$
$$-240 = 100t - 10t^2$$
$$-24 = 10t - t^2$$
$$t^2 - 10t - 24 = 0$$
$$(t-12)(t+2) = 0$$

$$t = 12 \ or \ t = -2$$

We reject the negative answer. The time of flight is therefore 12 s.

(d) $v = \frac{d}{t}$

$$86.6 = \frac{d}{12}$$

$$d = 1040\ m$$

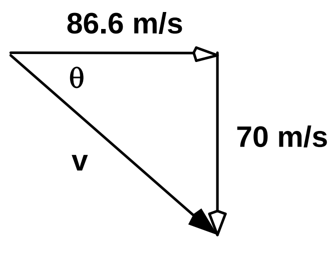
(e) On impact, the critter's velocity will have both a horizontal and a vertical component. The horizontal component is, of course, 86.6 m/s. We need to find the vertical component.

$u = +50 m/s$; $a = -10 m/s^2$; $d = -120 m$; $v = ?$

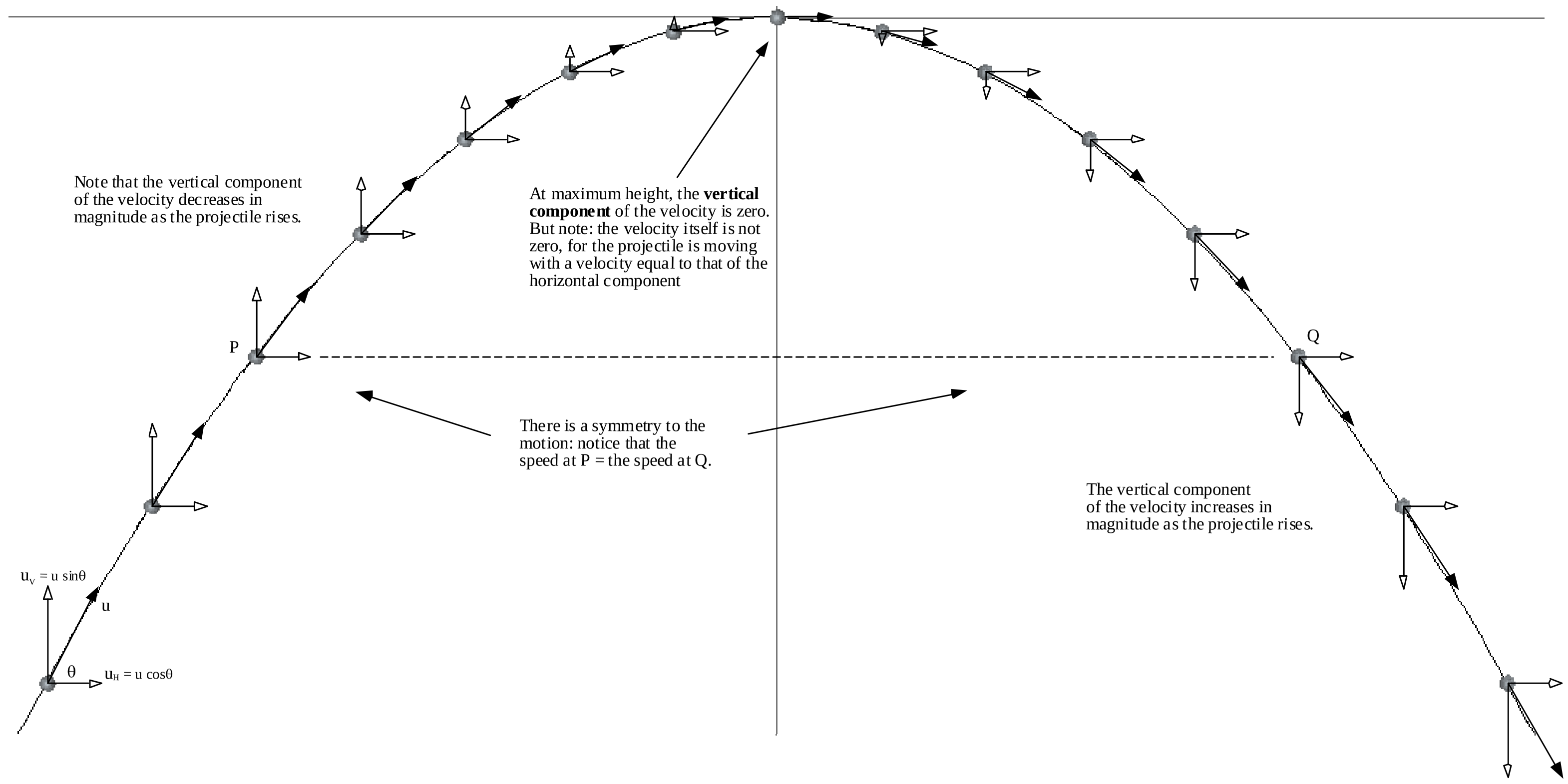
$$v^2 = u^2 + 2ad$$
$$v^2 = 50^2 + 2(-10)(-120)$$
$$v^2 = 2500 + 2400$$
$$v^2 = 4900$$
$$v = \pm\sqrt{4900}$$
$$v = \pm 70$$

Since it is moving downwards at impact, the appropriate answer is $v = -70\ m/s$.

The final velocity is the sum of the vertical and horizontal components.



So, $v = 111\ m/s$ at an angle of 39° below the horizontal.

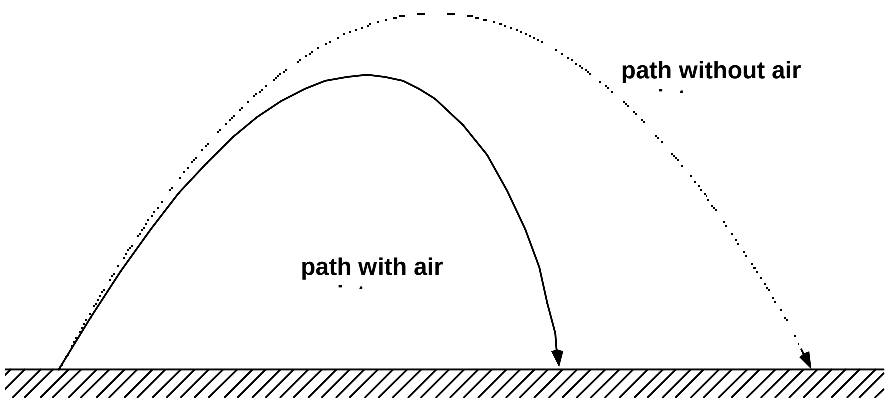

Note that the vertical component
of the velocity decreases in
magnitude as the projectile rises.
At maximum height, the **vertical**
component of the velocity is zero.
But note: the velocity itself is not
zero, for the projectile is moving
with a velocity equal to that of the
horizontal component
P
Q
There is a symmetry to the
motion: notice that the
speed at P = the speed at Q.
The vertical component
of the velocity increases in
magnitude as the projectile rises.
$u_V = u \sin\theta$
u
θ
$u_H = u \cos\theta$

AIR RESISTANCE

Air resistance is a nuisance (unless you are a parachutist). Being a frictional force, it is always in the opposite direction to the velocity of the object. In magnitude, it changes according to the speed of the object (it being greater at higher speeds). Since a projectile is always changing its velocity, the air resistance is also always changing, in both magnitude and direction. This means that projectile motion calculations that include air resistance are beyond the abilities of mere secondary school students. You are, however, required to understand the qualitative effects of air resistance. These can be summarised as follows:

- Air resistance always opposes the motion of the projectile.
- The magnitude of the air resistance increases as the speed of the projectile increases.
- The projectile transfers some of its kinetic energy to the air in the form of turbulence.

For an object projected from ground level, the path would be as follows:

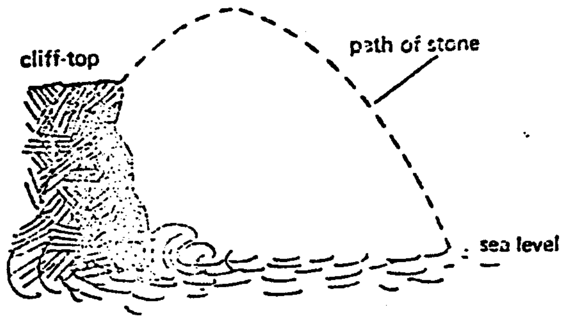


With air resistance, the following differences are apparent:

- The aesthetically pleasing symmetry no longer exists. The maximum height is reached beyond the mid-horizontal position.
- The angle of impact is greater that the angle of projection.
- Reduced horizontal displacement.
- Reduced vertical displacement.
- The final speed of the projectile will be less than the initial speed (due to transfer of energy to the air).

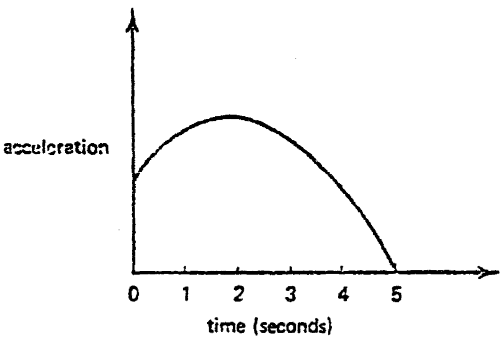
ADDITIONAL QUESTIONS ON PROJECTILE MOTION

QUESTION 22
A stone is thrown upwards from a cliff top and follows the path indicated in the following diagram.

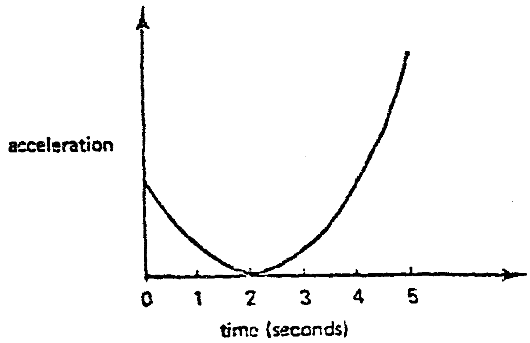


During its flight, the graph of the acceleration of the stone (neglecting air resistance) against time is best shown by which of the following diagrams.

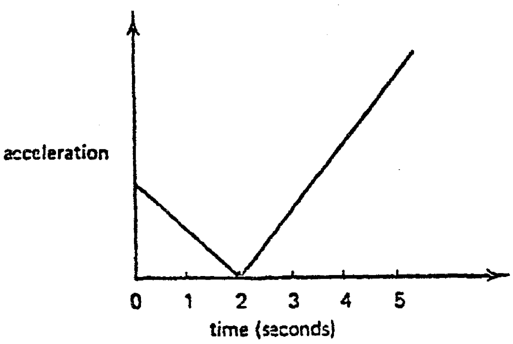
A



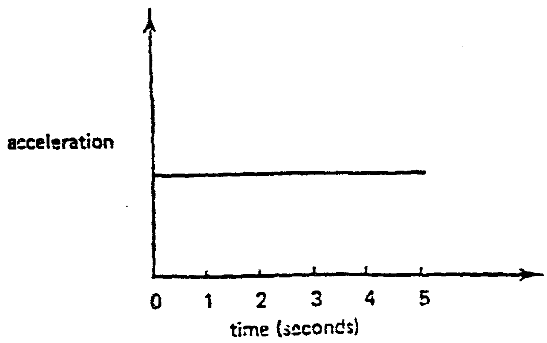
C



B



D



QUESTION 23

A boy standing on the tray of a truck traveling at $15\ m.s^{-1}$ throws a ball vertically upwards with a speed of $24\ m.s^{-1}$ and catches it again at the same level. What distance horizontally does the ball move while it is in the air?

Solution

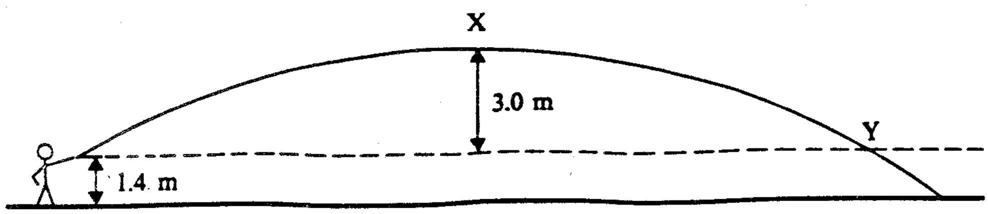
QUESTION 24
Two stones are thrown simultaneously in line and horizontally from the top of a cliff 245 m high with speeds 16 ms^{-1} and $64\ ms^{-1}$ respectively.

(a) How far apart will they strike the water?
(b) Find the speed of the first stone as it reaches the water.

Solution

The following information applies to Questions 25 to 29

A boy throws a ball of mass 0.20 kg an initial speed of $12\ ms^{-1}$ from a height 1.4 metres above the ground. It travels in a path as shown in the diagram, reaching a maximum vertical height of 3.0 metres above the starting point. X is the highest point the ball reaches.
Take $g = 10\ Nkg^{-1}$ and neglect air resistance.



QUESTION 25
What is the kinetic energy of the ball immediately after it leaves the boy's hand?

Solution

QUESTION 26
By how much does the potential energy at point X exceed the potential energy of the ball as it leaves the boy's hand?

Solution

QUESTION 27
What is the kinetic energy of the ball at point X?

Solution

QUESTION 28
What is the speed of the ball at point X?

Solution

QUESTION 29
What is the direction of the acceleration of the ball at point X?

A Horizontal, to the right.
B Horizontal, to the left.
C Vertically up.
D Vertically down.
E In the direction X to Y.
F No direction, as the acceleration is zero.

A “bowling machine” is designed to launch a cricket ball horizontally from a height of 2.45 m.

QUESTION 30
At what time after the launch does the ball first strike the ground?

Solution

QUESTION 31
What horizontal velocity must be given to the ball to ensure that it lands 17.5 m from the machine?

Solution

QUESTION 32

A stone is projected being $20\ ms^{-1}$ horizontally from a height of 180 m. Find:

(a) The time the stone takes to reach water.
(b) The distance from the base of the cliff at which it strikes the water.
(c) The speed at this distance.

Solution

A ball of mass 0.1 kg rolls on a horizontal table at $2\ ms^{-1}$. It hits the ground 0.4 seconds after rolling off the edge. Take $g = 10\ m.s^{-2}$.

QUESTION 33
What is the horizontal distance from the edge of the table to the point where it hits the ground?

Solution

QUESTION 34
What is the height of the table?

Solution

QUESTION 35
At what speed does the ball hit the ground?

Solution

QUESTION 36
At what angle with the horizontal does it hit the ground?

Solution

The ball bounces, and rises to a height of 0.5 m.

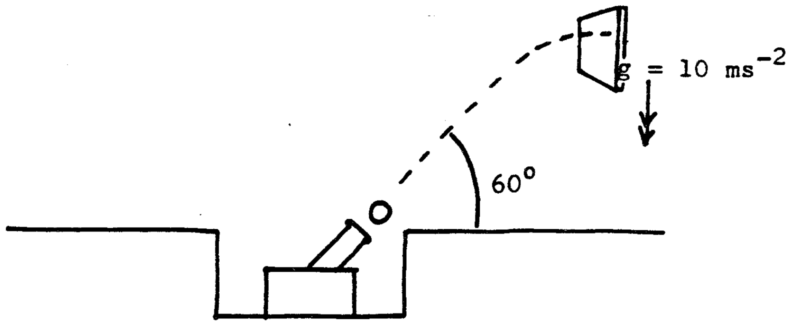
QUESTION 37
How much time will elapse between the first and the next time it hits the ground?

Solution

QUESTION 38
If, on the second bounce, it loses the same proportion of its kinetic energy as it did on the first bounce, how high will it rise after the second bounce?

Solution

A toy cannon fires a 5.0×10^{-3} kg ball with a velocity of $6.0\ ms^{-1}$ at 60° to the horizontal.



QUESTION 39
What is the maximum height that the ball will reach?

Solution

When the ball reaches maximum height it hits the vertical face of a slab of putty and it is embedded in it.

QUESTION 40
What is the magnitude of the horizontal impulse that the putty exerts on the ball?

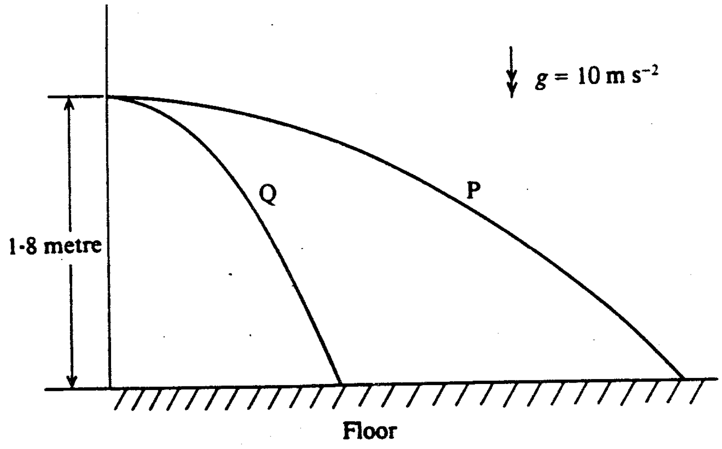
Solution

QUESTION 41
What is the value of the ratio?

$$\frac{\text{Kinetic energy of the ball just before hitting putty}}{\text{Initial kinetic energy of the ball.}}$$

Solution

Two projectiles *P* and *Q* each of mass 2.0 kg are given initial horizontal velocities of 5.0 and 3.0 ms^{-1} respectively, from a point 1.8 m above the floor. The path of each projectile is shown in the diagram. Assume air resistance is zero and take $g = 10 ms^{-2}$.



QUESTION 42
Calculate the kinetic energy, in joules, of *P* immediately before it strikes the floor.

Solution

QUESTION 43
Calculate the value of the ratio:

$$\frac{\text{Time of flight of } P}{\text{Time of flight of } Q}$$

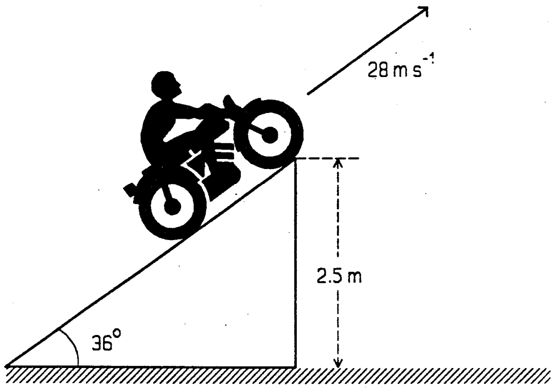
Solution

QUESTION 44
Calculate the horizontal distance traveled by *P* before it first strikes the floor.

Solution

A motorcyclist performs a ramp jumping stunt on the horizontal area at the Melbourne Showground. He does this by launching his motorbike from the end of the ramp 2.5 m above the ground at $28.0\ ms^{-1}$. The ramp is inclined at 36° to the horizontal. (Ignore the effects of air resistance).

Take $g = 10\ ms^{-2}$.



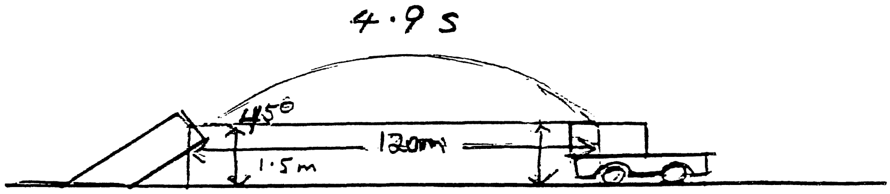
QUESTION 45
For how long was the motorcyclist airborne?

Solution

QUESTION 46
How far from the end of the ramp did the motorcycle and rider land?

Solution

A missile launcher fires a rocket at 45° to the horizontal from a height of 1.5 m above the ground. It hits a target vehicle 12 m away horizontally at the same height above the ground.



QUESTION 47
Find the horizontal component of the rocket's velocity as it leaves the launcher.

Solution

QUESTION 48
Calculate the magnitude of the velocity of the rocket at launch.

Solution

QUESTION 49
Find the highest point of the rocket above the ground (show working).

Solution

A ball is thrown from ground level at 30 ms^{-1} at an angle of 60° to the horizontal.
Air resistance is negligible ($g = 10\ ms^{-2}$).

QUESTION 50
What is the smallest value of the speed of the ball during its flight?

Solution

QUESTION 51
What is the maximum height reached by the ball?

Solution

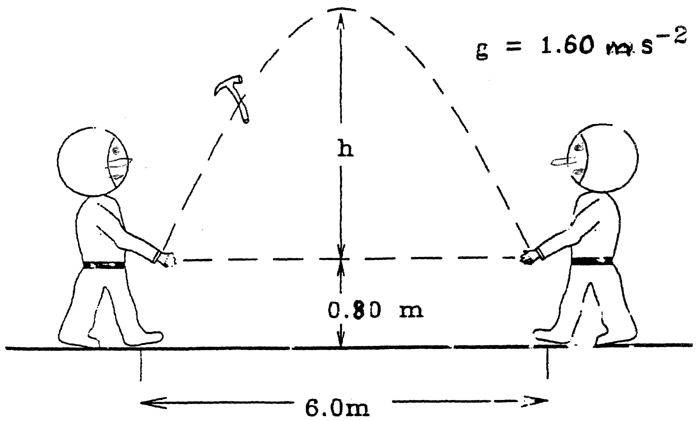
QUESTION 52
Which of the following is the change in velocity of the ball from just after its flight began, to just before it hit the ground?

A $15\ ms^{-1}$ forwards
B $15\ ms^{-1}$ backwards
C $60\ ms^{-1}$ forwards
D $60\ ms^{-1}$ backwards
E $30\ ms^{-1}$ upwards
F $30\ ms^{-1}$ downwards
G $42\ ms^{-1}$ upwards
H $42\ ms^{-1}$ downwards
I $52\ ms^{-1}$ upwards
J $52\ ms^{-1}$ downwards
K $60\ ms^{-1}$ upwards
L $60\ ms^{-1}$ downwards
M Zero
N None of (A) to (M)

Mark, Rosita and Raymond are astronauts of the future, working on a moon station.

Their advanced-technology space suits do not hinder their movements, so they are able to move with as much ease as they do on earth. The acceleration due to the moon's gravitational field near its surface is 1.60 ms^{-2} and there is no air resistance on the moon.

Mark throws Rosita a hammer, releasing it at a height of 0.80 m as shown in the diagram below. Rosita, standing 6.0 m away, catches it at the same height, 5.0 seconds after it is thrown.



QUESTION 53
What is the vertical component of the hammer's velocity just as it leaves Mark's hand?

Solution

QUESTION 54
What is the speed of the hammer when Rosita catches it?

Solution

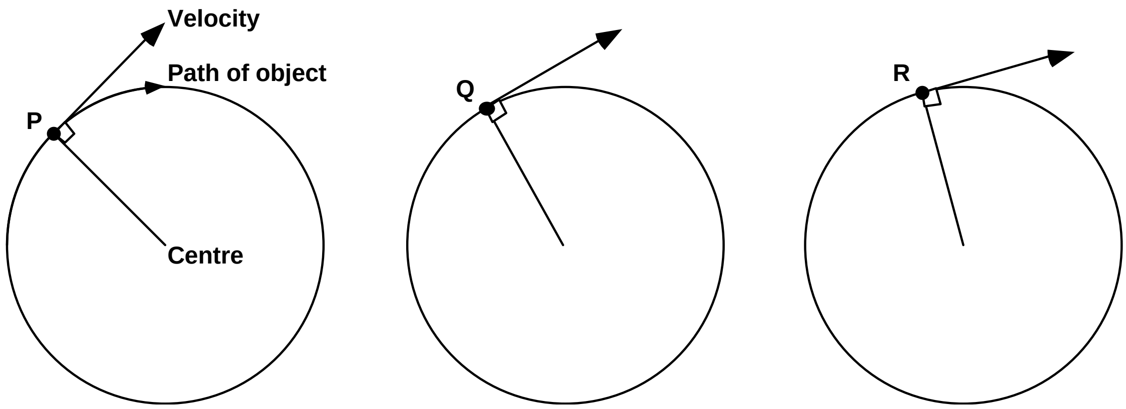
QUESTION 55
To what height (see the previous diagram) does the hammer rise?

Solution

CENTRIPETAL (CIRCULAR) MOTION

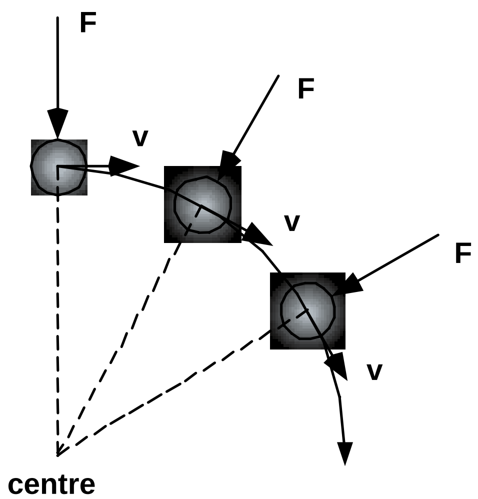
CIRCULAR MOTION AT CONSTANT SPEED

When an object moves along some path, its velocity vector at any given point is always a tangent to the path at that point. In the following diagram, the velocity vector for an object at point P is shown. As the object moves along the path, from point P to point Q and point R, etc, its velocity vector changes direction.



The changing velocity implies that a force is acting in order to bring about the change in velocity. An object can move along a circular path only if an external net force causes it to do so – if there were no net force, it would move in a straight line. (Remember Newton's First Law: An object will have constant velocity unless it is acted upon by some net external force).

If the speed around the circle is constant, this implies that there is never a component of the external net force acting in the direction of the velocity vector, i.e. the force must always be perpendicular to the velocity. Since the velocity is always tangential to the circle, the force must be directed radially, i.e. towards the centre of the circle.



The speed of the object can be found thus:

$$\text{speed} = \frac{\text{distance covered}}{\text{time taken}}$$

$$\text{speed} = \frac{\text{circumference}}{\text{period}}$$

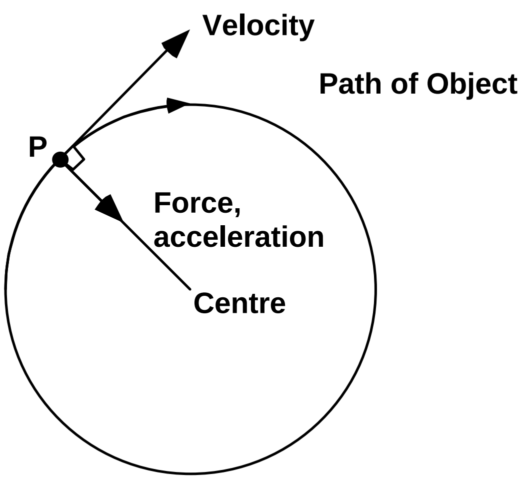
$$v = \frac{2\pi r}{T}$$

Even when moving at constant speed, the object is actually accelerating because it is changing its velocity. The acceleration is given by:

$$a = \frac{v^2}{r}$$

The direction of the acceleration is the same as that of the net force: Always towards the centre of the circle.

The net force that keeps the object moving in a circle is called the centripetal force, F_c.
(The acceleration of the object is called centripetal acceleration, a_c).



The magnitude of the force can be found by substituting $a_c = \frac{v^2}{r}$ into $F_{net} = ma$ and writing F_c instead of F_{net}. This gives:

$$F_c = m\frac{v^2}{r}$$

We can derive another set of formulae thus:

Substituting $v = \frac{2\pi r}{T}$ into $a_c = \frac{v^2}{r}$, we obtain:

$$a_c = \frac{4\pi^2 r}{T^2}$$

Substituting $a_c = \frac{4\pi^2 r}{T^2}$ into $F_{net} = ma$, we obtain:

$$F_c = m\frac{4\pi^2 r}{T^2}$$

EXAMPLE 11
A car is driven around a roundabout at a constant speed of 20 kmhr^{-1}. The roundabout has a radius of 7 m and the car has a mass of 1200 kg. What is the magnitude and direction of the acceleration of the car?

Solution

The first step is to convert the speed into metres/second. The conversion factor for converting kmhr^{-1} to ms^{-1} is 3.6.

$$v = \frac{20}{3.6} = 5.6\, ms^{-1}$$

$a = \frac{v^2}{r} = \frac{5.6^2}{7} = 4.5\ ms^{-1}$ The direction is towards the centre of the roundabout.

EXAMPLE 12
Two girls of similar mass are travelling side by side on a merry-go-round. Jenny is on the outer horse which is about 1 m further from the centre than Kathy's horse. Who would experience the greater centripetal force?

Solution

$$F_c = \frac{m4\pi^2 r}{T^2}$$

$F_c \propto r$ (T and m is the same for both girls) therefore Jenny experiences the greater force as her radius is the greater.

EXAMPLE 17
In a hammer throw competition an athlete swings a 6.0 kg hammer in a horizontal circular path. Calculate the horizontal component of tension in the handle if the hammer is moving at 25 ms^{-1} in a circle of 1.5 m.

Solution

$F_c = \frac{mv^2}{r} = \frac{6 \times 25^2}{1.5} = 2500N$ towards the centre.